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SOLUTION OF PROBLEMS IN NUMBER ONE.

SOLUTIONS of problems in No. 1 have been received as follows:

From Marcus Baker, 141, 142, 143 and 148; G. M. Day, 141, 142 and 143; Prof. H. T. Eddy, 149; Henry Gunder, 141, 142, 143, 144 and 148; Henry Heaton, 147 and 150; Prof. A. Hall, 149; Wm. Hoover, 143, 146 and 148; Christine Ladd, 141 and 142; W. L. Marcy, 141, 142, 143, 144, 148 and 150, also I to V inclusive, of Prof. Pratt's problems, (p. 186, Vol. III); Artemas Martin, 141, 146, 148 and 149; D. J. Mc. Adam, 143, 146 and 148; W. V. Mc. Knight, 144; J. Scheffer, 141, 142, 143 and 148; E. B. Seitz, 141, 142, 143, 144, 145, 146, 147, 148 and 149.

141. "Let ABC be any plane triangle, R the radius of its circumscribing circle, r' the radius of the escribed circle opposite A, p_1, p_2, p_3 the perpendiculars from the center of the circumscribing circle upon a, b and c. Show that $r' - R = p_2 + p_3 - p_1$."

SOLUTION BY W. L. MARCY, DEL NORTE, COLORADO.

The relation [Chauvenet, Trig., p. 78, (298)]

$$r \div R = -\cos A + \cos B + \cos C + 1$$

gives $\quad r - R = R(\cos C + \cos B - \cos A)$.

But $\quad R\cos C = p_3\,;\ R\cos B = p_2\,;\ R\cos A = p_1$.

Substituting, there results

$$r - R = p_3 + p_2 - p_1.$$

142. "In a plane triangle ABC, AO' is drawn bisecting the angle A; from O', the center of the escribed circle, a perpendicular to AO' is drawn, meeting AB produced in T', and from T' a perpendicular to AB is drawn, meeting AO' produced in O''; with O'' as center and $O''T'$ as radius a circle is described; prove that this circle is tangent to the circle circumscribed about the triangle ABC."

SOLUTION BY G. M. DAY, LOCKPORT, N. Y.

Put $O''T' = R'$, radius of circumscribed circle $= R$, and let its centre be denoted by O. Let $OO'' = d$, and $r' =$ radius of escribed circle. Then is

$$d^2 = R^2 + R'\text{cosec}^2\tfrac{1}{2}A - 2RR'\text{cosec}\,\tfrac{1}{2}A\cos\tfrac{1}{2}(B - C). \;.\;.\;.\;(1)$$

If the circles whose centres are O and O'' are tangent, we must have

$$d^2 = (R + R')^2 \; . \; . \; . \; . \; (2), \qquad R' = r' \sec^2 \tfrac{1}{2}A. \; . \; . \; . \; . \; (3)$$

Combining (1), (2) and (3) we obtain

$$\frac{r'}{2R \, \sin^2 \frac{1}{2}A} = 1 + \frac{\cos \frac{1}{2}(B - C)}{\sin \frac{1}{2}A} . \; . \; . \; . \; . \; . \; . \; . \; . \; (4)$$

But each member of (4) $= 2 \cos \frac{1}{2}B \cos \frac{1}{2}C \div \sin \frac{1}{2}A$, $\therefore$ the proposition is proved.

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143. "Required the area of the triangle formed by the tangent to the curve whose equation is $x^{2/3} + y^{2/3} = r^{2/3}$ and the axes of x y."

SOLUTION BY HENRY GUNDER, NORTH MANCHESTER, IND.

Let a and b be the coordinates of the curve, and x and y, of the tangent line.

The equation to the tangent is

$$y - b = \frac{db}{da}\left(x - a\right) \ldots (1). \quad \text{From } a^{2/3} + b^{2/3} = r^{2/3}, \quad \frac{db}{da} = -\frac{b^{1/3}}{a^{1/3}} \ldots (2)$$

Therefore, by substitution, (1) becomes $y - b = \dfrac{b^{1/3}}{a^{1/3}}\left(a - x\right). \; . \; . \; . \; . \; . \; (3)$

Making x and y each $= 0$, in succession, in (3) we get,

$$y = a^{1/3} r^{2/3} \; . \; . \; . \; (4), \quad x = b^{1/3} r^{2/3}. \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (5)$$

Hence the area $= \frac{1}{2}\sqrt[3]{(abr^4)} = \frac{1}{2}r\sqrt[3]{(abr)}$.

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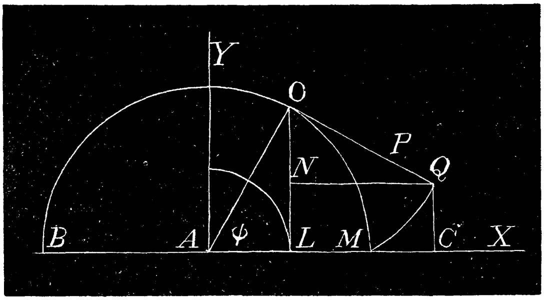
144. "A cylindrical post, diameter d, stands perpendicularly on a level plane with a rope, diameter d' and length l, attached to the bottom of the post and lying stretched on the plane. How far will a person walk who takes hold of the end of the rope and winds it around the post by walking around it always keeping the rope parallel to the plane and also keeping the coils on the post at a distance of $3d'$ from each other?"

SOLUTION BY W. V. MC. KNIGHT, PARKSVILLE, KENTUCKY.

Let BOM represent the semi-base of the post, A, its center, OQ, a part of the rope, unwound, and MQ the projection upon the plane, of the curve described by the end of the rope in being wound to M. Because one spire of the rope about the cylinder is greter than the circumference of the cylinder, the radius of curvature of any point of the required curve is greater than the radius of curvature of the corresponding point of the arc of an involute to the base of the post; let v $(= PQ)$ represent this difference.

Draw OL and OC perpendicular to the axis of X, and QN perpendicular to OL. We have $AO = \frac{1}{2}(d + d')$, which put $= a$, and let $AC = x$, $QC = y$, curve $MQ = z$ and angle $OAM = \varphi$. Let s represent the difference between the length of one spire and the circumference of the post; then is $2a\pi : a\varphi :: s : v$; Therefore $v = s\varphi \div 2\pi$, and $OQ = a\varphi + s\varphi \div 2\pi = (a + s \div 2\pi)\varphi = h\varphi =$ the radius of curvature for the point Q. We have



$$AL + NQ = x = a\cos\varphi + h\varphi\sin\varphi, \;.\;.\;.\;.\;.\;.\;.\;.\; (1)$$

$$OL - NO = y = a\sin\varphi - h\varphi\cos\varphi. \;.\;.\;.\;.\;.\;.\;.\;.\; (2)$$

Differentiating (1) and (2), squaring, and adding them together, observing that $dz^2 = dx^2 + dy^2$, we have, putting $h - a = k$,

$$dz = \sqrt{(h^2\varphi^2 + k^2)}d\varphi. \;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\;.\; (3)$$

Integrating (3) we find

$$z = \tfrac{1}{2}\varphi\sqrt{(h^2\varphi^2 + k^2)} + (k^2 \div 2h)\log[\sqrt{(h^2\varphi^2 + k^2)} + h\varphi] + C.$$

When $z = 0$ $\varphi = 0$; $\therefore$ $C = -(k^2 \div 2h)\log k$; and when $h\varphi = l$, $\varphi = l \div h = c$ say. Substituting these values for C and φ, we find

$$z = \tfrac{1}{2}c\sqrt{(h^2c^2 + k^2)} + (k^2 \div 2h)\log\left\{ [\sqrt{(h^2c^2 + k^2)} + hc] \div k \right\},$$

which is the distance walked by the man in winding the rope about the post as required, provided he is able to keep the rope horizontal while walking on a level plane.

[As z is the projection upon the plane of the curve described by the end of the rope, if the person is supposed to ascend as the end of the rope ascends, in winding around the post, the distance walked, in that case is a curve of double curvature and is represented by the projection of z upon the surface of an hyperboloid.—In all the other solutions, of this question, that have been received, the curve found is the involute of a circle whose circumference is the length of one spire aroud the post, and is therefore a slightly different curve from the one required. As the distance between the spires, if taken perpendicular to the rope, and if in general measurable on the surface of the post, is a curve, we infer that by "a distance of $3d'$ from each other" is meant a *vertical* distance of $3d'$. On this supposition the value of h, as determined by Mr Seitz, is

$$\frac{\pi(d+d')\left\{ 3d'^2 + \pi(d+d')\sqrt{[\pi^2(d+d') + 8d'^2]} \right\}}{\pi^2(d+d') - d'^2}.$$

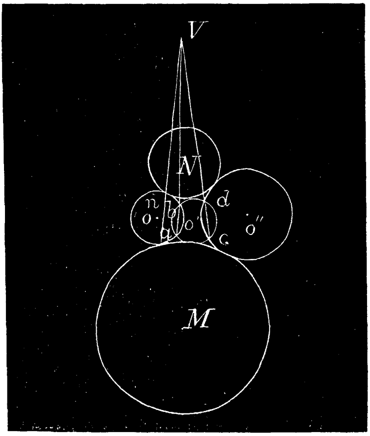
Ed.]

145. "The product of the lengths of tangents from the radical center of three circles on any pair of circles through the intersections of the given circles is equal to the product of the lengths of tangents from the same point on any pair of circles tangent to the given circles."

SOLUTION BY E. B. SEITZ, GREENVILLE, OHIO.

Let O, O', O'' be the three given circles, O, O' intersecting in a, b, and O', O'' in c, d, and let M, N be a pair of circles tangent to the three given circles.

Produce $a\,b$ and $c\,d$ till they meet in V, which is the radical centre of O, O', O''. Since the circles O and O' touch the circles M and N, and the contacts are of the same kind, the radical axis of O and O' passes through the external center of similitude of M and N; and for the same reason the radical axis of O' and O'' passes through the same point. Therefore, V is the external center of similitude of M and N.



Let T denote the length of the tangent from V to any circle through a, b or c, d, and T', T'' the lengths of the tangents from V to the circles M and N. Then by well known principles we have $T^2 = Va.Vb$, and $T' \times T'' = Vm\,.\,Vn = Va\,.\,Vb$; $\therefore$ $T^2 = T' \times T''$.

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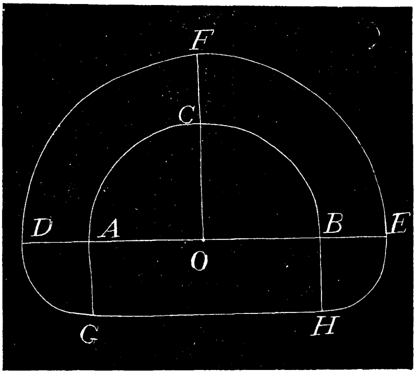
146. "Given a semicircle and a circle, place the latter so that it will cut the former; what is the probability that its center will fall within the former?"

SOLUTION BY ARTEMAS MARTIN, ERIE, PA.

Let $A\,B\,C$ be the given semicircle, radius R. With same center O describe a semicircle DEF, radius $R+r$, r being the radius of the given circle. With centers A and B, and radii r, describe the quadrants DG and EH.

If the center of the circle be placed within the boundary $DFEHG$ it will cut the semicircle.



Area $ABC = \frac{1}{2}\pi R^2$, area $DEF = \frac{1}{2}\pi(R + r)^2$, area $AGBH = 2Rr$, and area ADG = area BEH = $\frac{1}{4}\pi r^2$. Hence

$$p=\frac{\frac{1}{2}\pi R^2}{\frac{1}{2}\pi r^2+\frac{1}{2}\pi(R+r)^2+2Rr}=\frac{\pi R^2}{\pi(R^2+2Rr+2r^2)+4Rr}.$$

If $R=r$, $p=\pi\div(5\pi+4)$.

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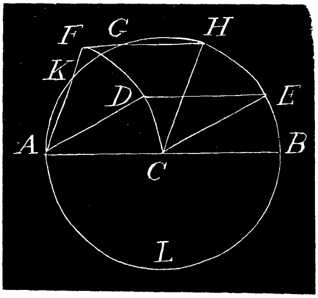
147. "Two rods of equal length have their middle points connected by a string of half the length of one of the rods. If they be thrown on a level floor, what is the chance of their crossing?"

SOLUTION BY E. B. SEITZ, GREENVILLE, OHIO.

For each position that one of the rods may take, the center of the other may fall anywhere on the surface of the circle described on the first as diameter, and the second may make any angle from 0 to $\frac{1}{2}\pi$ with the first. Hence we may regard one of the rods fixed in position on the floor, and consider all the positions the other may take.



Let AB represent the first rod, C its center, $AGHBL$ the circle on AB, and let θ = the angle which the second rod makes with the first. Draw AD and AF each equal to AC, making $\angle DAB<\frac{1}{3}\pi$, $\angle FAB>\frac{1}{3}\pi$; draw DE and FH parallel to AB, and join CE and CH.

When $\angle DAB=\theta$, the second rod will cross the first if its center falls anywhere on the surface $ADEB$; and when $\angle FAB=\theta$, the second rod will cross the first, if its center falls anywhere on the surface $AKGHB$. Put $AC=1$, $\angle DAB$, or $FAB=\theta$, area $ADEB=u_1$, area $AKGHB=u_2$. Then $u_1=\frac{1}{2}\theta+\sin\theta$, and $u_2=2\theta-\frac{1}{2}\pi+2\sin\theta\cos\theta$.

Hence, doubling, since the center of the second rod may fall on either side of AB, we find for the chance required.

$$p=\frac{2\int_0^{\frac{1}{3}\pi}u_1d\theta+2\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi}u_2d\theta}{\int_0^{\frac{1}{2}\pi}\pi d\theta}$$

$$=\frac{4}{\pi^2}\int_0^{\frac{1}{3}\pi}(\tfrac{1}{2}\theta+\sin\theta)d\theta+\frac{4}{\pi^2}\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi}(2\theta-\tfrac{1}{2}\pi+2\sin\theta\cos\theta)d\theta$$

$$=\frac{1}{3}+\frac{3}{\pi^2}.$$

148. "A tortoise, whose shell is circular, radius a, is moving in a **straight** line at the uniform speed of m feet per minute, and a fly is running around on the edge of its shell at the uniform rate of n feet per minute.

Required the equation to the curve the fly describes in space."

SOLUTION BY D. J. MC. ADAM, WASHINGTON, PA.

Take the line of motion of the tortoise as axis of x. Let the fly be initially in the axis. At the time t suppose a line drawn from the fly to the centre of the tortoise to make an angle I with the axis of x. Then

$$y = a \sin I \ldots (1), \qquad x = mt + a - a\cos I \ldots (2), \qquad nt = aI \ldots (3)$$

Eliminating t and I in (2), and putting $m \div n = r$,

$$x = raI + a - a\sqrt{(1 - \sin^2 I)} = ra \sin^{-1}(y \div a) + a - \sqrt{(a^2 - y^2)}.$$

The curve belongs to the family of cycloids. If $m=n$ it is the common cycloid.

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149. "Prove that

$$\int_0^{\frac{a}{2}} \frac{dx}{\sqrt{(2ax - x^2)}\sqrt{(a^2 - x^2)}} = \frac{2}{3a} F\left(c \frac{\pi}{2}\right), \text{ where } c = \tfrac{1}{3}."$$

SOLUTION BY PROF. H. T. EDDY, CINCINNATI, OHIO.

$$u = \int_0^{\frac{1}{2}a} \frac{dx}{\sqrt{(2ax - x^2)}\sqrt{(a^2 - x^2)}}.$$

Let $$\frac{x}{a} = \frac{\tan^2\varphi}{1 + \sec^2\varphi}, \quad \therefore u = \frac{1}{a}\int_0^{\tan^{-1}2} \frac{d\varphi}{\sqrt{(1 - \frac{3}{4}\sin^2\varphi)}}.$$

Again, let

$$\frac{\sin 2\varphi}{c_1 + \cos 2\varphi} = \frac{2\tan\varphi}{1 + c_1 - (1 - c_1)\tan^2\varphi} = \tan\theta, \text{ where } c_1^2 = \tfrac{3}{4},$$

$$\therefore \quad u = \frac{2}{3a}\int_0^{\frac{1}{2}\pi} \frac{d\theta}{\sqrt{(1 - \frac{1}{9}\sin^2\theta)}} = \frac{2}{3a} F\left(c, \frac{\pi}{2}\right), \text{ where } c = \tfrac{1}{3}.$$

SOLUTION BY E. B. SEITZ.

Let $x = \frac{1}{2}a(1 - \sin\theta)$. Then $2ax - x^2 = \frac{1}{4}a^2(1 - \sin\theta)(3 + \sin\theta)$, $a^2 - x^2 = \frac{1}{4}a^2(1 + \sin\theta)(3 - \sin\theta)$, $dx = -\frac{1}{2}a\cos\theta\, d\theta$;

$$\therefore \quad \int_0^{\frac{1}{2}a} \frac{dx}{\sqrt{(2ax - x^2)}\sqrt{(a^2 - x^2)}} = \frac{2}{3a}\int_0^{\frac{1}{2}\pi} \frac{d\theta}{(\sqrt{1 - c^2\sin^2\theta})} = \frac{2}{3a} F\left(c, \frac{\pi}{2}\right).$$

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150. "An underwriter insures three vessels, the first an iron steamer, the second a steamer not of iron and the third a sailing vessel, at \$20,000, \$15,000, and \$10,000, respectively. One of them is known to have been

burned at sea; and three persons, *A*, *B*, *C*, whose respective veracities are $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, report as follows: *A*, that the lost vessel was an iron steamer; *B*, that it was not a sailing vessel; and *C*, that it was a sailing vessel. Required the expectation of loss to the underwriter, the *a priori* probability of destruction by fire being twice as great in case of a steamer as of a sailing vessel."

SOLUTION BY HENRY HEATON, B. S., DES MOINES, IOWA.

Before receiving the testimony of either *A*, *B* or *C* the chances of burning of the different vessels were as 2, 2, and 1; after *A*'s testimony they were as 6, 2, and 1; after *B*'s, as 24, 8, and 1, and after *C*'s, as 24, 8, and 5, taking them in the order in which they are named in the problem.

Hence the chances of the burning of the different vessels are $\frac{24}{37}$, $\frac{8}{37}$ and $\frac{5}{37}$; and the expectation of loss is $\frac{24}{37}$ of \$20000 $+$ $\frac{8}{37}$ of \$15000 $+$ $\frac{5}{37}$ of \$10000 $=$ \$17567.56$\frac{28}{37}$.

PROBLEMS.

151. By A. W. Mason, Cedar Falls, Iowa.—What is the altitude of the maximum cylinder which can be inscribed in a given paraboloid?

152. By G. M. Day, Lockport, N. Y.— Find the surface of a right conoid with a circular base.

153. By J. B. Mott, Neos., Mo.—Prove that if $(1+0\times\frac{1}{6}x\sqrt{-1})^{1\div 0}-(1-0\times\frac{1}{6}x\sqrt{-1})^{1\div 0}=\sqrt{-1}$ ... (1), $x=[(2\sqrt{-1})\div 0][(1+\sqrt{-1})^0-(1-\sqrt{-1})^0]$ ... (2); and find the real approximate value of x.

154. By Prof. O. Pratt, Sen., Salt Lake City, Utah. — Find a general logarithmic theorem for the differentiation of

$$u=z^{x_1^{x_2^{\cdot^{\cdot^{\cdot^{x_n}}}}}}, \quad z,\ x_1,\ x_2, \text{ \&c., being any functions}$$

of one variable as x.

155. By Prof. C. Bancroft, Hiram, Ohio. — To find the least distance (in miles on the earth's surface) between two places given by latitude and longitude, taking into account the polar compression.

156. By Prof. Johnson. — In a determinant of the nth degree the elements of the principal diagonal consist of units, and of the remaining elements those in the first column are each equal to a, those in the second column each equal to b and so on. Evaluate the determinant.